Physics 1AH CheatSheet

DO SANITY CHECKS!!!

Trig

$$2\sin^2\theta = 1 - \sin 2\theta$$
$$2\cos^2\theta = 1 + \cos 2\theta$$
$$\cos\theta \approx 1 - \theta^2/2$$
$$\sin\theta \approx \theta$$

Cartesian to Polar

$$x = R\cos\theta$$
$$y = R\sin\theta$$
$$R^2 = x^2 + y^2$$
$$\theta = \arctan(\frac{x}{y})$$
$$\hat{r}(t) = \cos\theta(t)\hat{i} + \sin\theta(t)\hat{j}$$
$$\hat{\theta}(t) = -\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}$$
$$\hat{i} = \cos\theta(t)\,\hat{r}(t) - \sin\theta(t)\,\hat{\theta}(t)$$
$$\hat{j} = \sin\theta(t)\,\hat{r}(t) + \cos\theta(t)\,\hat{\theta}(t)$$

Polar Coordinates

$$\boxed{\tilde{\mathbf{a}} = (\ddot{\mathbf{r}} - \mathbf{r}\omega^{\mathbf{2}})\hat{\mathbf{r}} + (\mathbf{r}\ddot{\theta} + \mathbf{2}\dot{\mathbf{r}}\dot{\theta})\hat{\theta}}$$
$$\vec{v} = \dot{r}\hat{r} + r\omega\hat{\theta}$$

Iris Method

First build a ring that has width equal to arclength

$$dA = 2\pi R\sin\theta \cdot Rd\theta$$

if the total subtends angle α

$$\int_0^{\frac{\alpha}{2}} 2\pi R\sin\theta \cdot Rd\theta = 2\pi R^2(1 - \cos\frac{\alpha}{2})$$

to find the force, for example gravitational

$$\boxed{dF = \frac{GMdm}{r^2}}$$
$$dm = \rho dV$$
$$dV = dAdr$$

the area changes as a function of angle and radius
there is also a factor of $\cos\theta$ so we only analyze
the force in the direction we're interested in

$$dF = \boxed{-\frac{GMdAdr}{r^2}\cos\phi}$$
$$= -\frac{GM\rho(2\pi R\sin\theta\cos\phi \cdot Rd\theta)\cdot dr}{r^2}$$
$$\int_0^F dF' = -2GM\pi \cdot \int_{R_i}^{R_o} \rho R^2 \frac{dr}{r^2} \cdot \int_0^{\frac{\alpha}{2}} \sin\theta\cos\phi d\theta$$

Simple Harmonic Motion

if the equation for the general solution is in the form of

$$\boxed{\ddot{x} = -kx}$$

it is SHM

$$\boxed{x(t) = C_1\cos(\omega t) + C_2\sin(\omega t)}$$
$$\boxed{A\sin(\omega t + \phi)}$$

where $\omega = \sqrt{\frac{k}{m}}$ generally

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Momentum

$$F = \frac{dP}{dt}$$

Momentum Flux

$$\boxed{F = \dot{P_{in}} - \dot{P_{out}}}$$
$$\boxed{\dot{P} = \vec{J}\cdot\vec{A}}$$
$$\vec{J} = \rho v^2\vec{v}$$

Rocket Equation

$$u\frac{dm}{dt} = M\frac{dv}{dt}, \quad M = M_o + \frac{dm}{dt}t$$
$$F + u\frac{dm}{dt} = M\frac{dv}{dt}$$
$$P_i = Mv$$
$$P_f = (M + dm)v_f$$

Energy

$$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = \int_{x_a}^{x_b} F(x)dx = \Delta KE$$
$$\boxed{W = \int_C \vec{F}\cdot d\vec{r}}$$
$$\boxed{F_x = -\frac{dU}{dx}}$$
$$\boxed{U = -W}$$
$$U_g = -\frac{GMm}{r}$$

Power

$$\boxed{P = \frac{dW}{dt}}$$
$$P = Fv_{avg}$$

Integrals and Derivatives

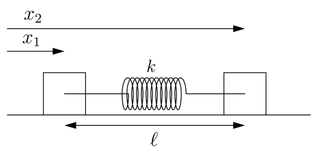
Derivatives

$$\frac{d}{dx}(log_a x) = \frac{1}{xlna}$$
$$\frac{d}{dx}(a^u) = a^x(lna)du$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Integrals

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\arccos(\frac{u}{a})^{-1} + C$$
$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin(\frac{u}{a}) + C$$
$$\int \frac{1}{a^2 + u^2} = \frac{1}{a}\arctan(\frac{u}{a}) + C$$
$$\int \tan u du = -\ln|\cos u| + C$$
$$\int \cot u du = \ln|\sin u| + C$$
$$\int \sec u du = \ln|\sec u + \tan u| + C$$
$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

Two masses and a spring



The two equations of motion for m_1 and m_2 respectively are

$$m\ddot{x}_1 = k(x_2 - x_1 - l)$$
$$m\ddot{x}_2 = k(x_2 - x_1 - l)$$
$$m(\ddot{x}_2 - \ddot{x}_1) = -2k(x_2 - x_1 - l)$$

substituting

$$u = x_2 - x_1$$
$$\ddot{u} = \ddot{x}_2 - \ddot{x}_1$$

We then get

$$m\ddot{u} = -2k(u - l)$$
$$m\ddot{u} = -2ku + 2kl$$
$$\ddot{u} + \frac{2k}{m}u = 2kl$$

We can see this can be represented as the differential equation

$$u = C_1 \sin\omega t + C_2 \cos\omega t + y_p$$

Where $\omega = \sqrt{\frac{2k}{m}}$

$$\dot{u} = C_1\omega\cos\omega t + C_2 \ldots$$
$$\dot{u}(0) = v_0$$
$$\dot{x}_1 - \dot{x}_2 = v_0 \cos\omega t$$

By conservation of momentum

$$\dot{x}_1 = v_0 - \dot{x}_2$$
$$v_0 - 2\dot{x}_2 = v_0 \cos\omega t$$
$$\dot{x}_2 = \frac{v_0}{2}(1 - \cos\omega t)$$
$$\dot{x}_1 = \frac{v_0}{2}(1 + \cos\omega t)$$